

# The black hole quantum atmosphere

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Ever since the discovery of black hole evaporation, the region of origin of the radiated quanta has been a topic of debate. Recently it was argued by Giddings that the Hawking quanta originate from a region well outside the black hole horizon by calculating the effective radius of a radiating body via the Stefan–Boltzmann law. In this paper we try to further explore this issue and end up corroborating this claim, using both a heuristic argument and a detailed study of the stress energy tensor. We show that the Hawking quanta originate from what might be called a quantum atmosphere around the black hole with energy density and fluxes of particles peaked at about  $4M$ , running contrary to the popular belief that these originate from the ultra high energy excitations very close to the horizon. This long distance origin of Hawking radiation could have a profound impact on our understanding of the information and transplanckian problems.

## I. INTRODUCTION

The discovery of Hawking radiation [1] changed our perspective towards black holes, giving us a deeper insight about the microscopic nature of gravity. At the same time, within the semi-classical framework, the current understanding of such process still leaves open several issues. Of course, a well known unresolved problem of black hole physics is the information loss paradox [2–4], i.e. the apparent incompatibility between the complete thermal evaporation of a black hole endowed with an event horizon and unitary evolution as prescribed by quantum mechanics.

For restoring unitarity of Hawking radiation and addressing the information loss problem correctly, it is important (among other things) to know from where the Hawking quanta originate. For example, if one assumes a near horizon origin of the Hawking radiation, then one way to restore unitarity is by conjecturing some sort of UV-dependent entanglement between partner Hawking quanta which would enable the late time Hawking flux to retrieve the information in the early stages of the evaporation process. Such scenario seems to lead to the so called “firewall” argument as the conjectured lack of maximal entanglement between the Hawking pairs makes the near horizon state singular and eventually demands some drastic modification of the near horizon geometry [5]. On the other hand, if one believes in a longer distance origin of the Hawking quanta, some effect must be operational at a larger scale for restoring unitarity rather than near the horizon, avoiding the “firewall”.

A similar open issue is the transplanckian origin of Hawking quanta. Hawking’s original calculation indicates that the quanta originate near the black hole horizon in a highly blue-shifted state requiring an assumption on the UV completion of the effective field theory used for the computation and on the lack of back-reaction on

the underlying geometry<sup>1</sup>. While it was debated for a while if Hawking quanta could originate initially, during the star collapse, and later released over a very long time, it was convincingly argued in [8] that this cannot be the case if an event horizon indeed forms. This leads to the conclusion that the Hawking quanta are generated in a region outside the horizon. A conclusion corroborated by studies of the Hawking modes correlation structure where it was shown that mode conversion happens over a long distance from the horizon [9]. A more recent claim in this direction, based on calculating the size of the radiating body via the Stefan–Boltzmann law, showed that the Hawking quanta originate in a near horizon quantum region, a sort of black hole “*atmosphere*” [10]. It is a well known fact that the typical wavelength of the radiated quanta is comparable to the size of the black hole, so one might think that the point particle description is not very accurate. However, as measured by a local observer near the horizon, the wavelength is highly blue-shifted when traced back from infinity to the horizon, thus validating the point particle description.

The Hawking process can be explained heuristically as well, for example via a tunnelling mechanism where the particle tunnels out of the horizon or the anti particle (propagating backwards in time) tunnels into the horizon and as a result of this we get the constant Hawking flux at infinity [11]. Alternatively, one popular picture is to imagine that the strong tidal force near the black hole horizon stops the annihilation of the particle and anti-particle pairs that are formed spontaneously from the vacuum. Once the antiparticle is “hidden” within the black hole horizon, having a negative energy effectively, the other particle can materialise and escape to infinity [12, 13].

In this paper we shall explicitly make use of this latter heuristic picture as well as of a full calculation of the stress energy tensor in 1+1 dimensions. We shall see that both methods seem to agree in suggesting that the

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<sup>1</sup> See, for instance, [6, 7] for a black hole evaporation analysis where these issues can be addressed in a quantum gravity context.

Hawking quanta originate from the black hole *atmosphere* and not from a region very close to the horizon. In section II, based on the heuristic picture of Hawking radiation described above and invoking the uncertainty principle and tidal forces, we show that most of the contribution to the radiation spectrum comes from a region far away from the horizon. In section III we further strengthen our claim by a detailed calculation of the renormalized stress energy tensor, which indicates a similar result.

## II. A GRAVITATIONAL SCHWINGER EFFECT ARGUMENT

One ingredient of our heuristic argument to identify a quantum atmosphere outside the black hole horizon, where particle creation takes place, is the uncertainty principle. However, the use of the uncertainty principle alone, as originally suggested by Parker [14], does not contain any physically relevant information about the location of particle production and why smaller black holes should be hotter. Indeed, the uncertainty principle in this case provides a rough estimate of the region of particle production as inversely proportional to the energy of the Hawking quanta when they are produced, but it does not take into account any dynamical mechanism to estimate the probability of spontaneous emission.

Thus one can improve this argument by invoking a physical process of creation of the Hawking quanta and using the uncertainty principle as a complementary tool to estimate the region of origin of the quanta. In this section, we try to achieve this goal by relying on tidal forces.

Let us then consider a situation where a virtual pair, consisting of a particle and anti-particle, pops out of the vacuum spontaneously for a very short time interval and then annihilates itself. In the Schwinger effect [15] a static electric field is assumed to act on a virtual electron-positron pair until the two partners are torn apart once the threshold energy necessary to become a real electron-positron pair is provided by the field. Energy is conserved due to the fact that the electric potential energy has opposite sign for partners with opposite charge. However, in its gravitational counterpart a priori only vacuum polarisation can be induced by a static field in the absence of an horizon.

In fact, only in the presence of the latter one has both the characteristic peeling structure of geodesics (diverging away from the horizon on both its sides) as well as the presence of an ergoregion behind it.<sup>2</sup> The presence of an ergoregion is crucial for energy conservation as it

allows for negative energy states given that in it the norm of the timelike Killing vector, with respect to which we compute energy, changes sign.

Indeed, if a Schwinger-like process takes place near the black hole horizon, due to the tidal force of the black hole and the peeling of geodesics, the pair can get spatially separated and one partner can enter the black hole horizon following a timelike or null curve with negative energy while the other particle can escape to infinity and contribute to the Hawking flux. In this picture, we are implicitly assuming that virtual particles in the vicinity of a black hole horizon move along geodesics when they are just about to go on-shell.

Therefore, the physical scenario we want to envisage is that of a particle-antiparticle pair pulled apart by the black hole tidal force outside the horizon until they go on-shell as one of them reaches the horizon<sup>3</sup> located at  $r_s = 2GM/c^2$  (actually an infinitesimal distance inside it so that the geodesic motion will drag it further inside) while the other particle is at a radial coordinate distance  $r = r_*$ . Once on-shell, the outgoing particle eventually reaches infinity and contributes to the Hawking spectrum. In order to do so though, it has to be created with an energy corresponding to the energy of the Hawking quanta at a distance  $r_* > r_s$  from the center of the black hole as measured by a local static observer; this can be reconstructed by noticing that

$$\omega_r = \frac{\omega_\infty}{\sqrt{g_{00}}}, \quad (1)$$

where  $\omega_\infty$  is the energy at infinity and we are using the  $(+, -, -, -)$  signature. At infinity, the thermal spectrum of Hawking radiation gives

$$\omega_\infty = \gamma \frac{k_B T_H}{\hbar}, \quad (2)$$

where the Hawking temperature for a black hole of mass  $M$  reads  $k_B T_H = \frac{\hbar c^3}{8\pi G M}$ . Thus, we get

$$\omega_\infty = \gamma \frac{c^3}{8\pi G M} \quad (3)$$

and

$$\omega_r = \gamma \frac{c}{4\pi r_s} \frac{1}{\sqrt{1 - \frac{r_s}{r}}}, \quad (4)$$

<sup>2</sup> This is strictly true only for non-rotating black holes, for rotating ones the ergoregion lies outside of the horizon allowing for the classical phenomenon of superradiance. However, the quantum emission still requires the peculiar peeling structure of geodesics typical of the horizon.

<sup>3</sup> One could also consider the case where the ingoing particle tunnels through the horizon and goes on-shell well inside the horizon (as e.g. suggested by the results of [9]); however, since in our analysis below we are interested in the tidal force as computed in the outgoing particle rest frame, this should not affect the final expression for the force. Thus, from the point of view of an outside static observer, the work done by the gravitational field on the pair (in our heuristic derivation) is insensitive to the exact location where the ingoing particle becomes real.

where  $\gamma$  is a numerical factor spanning the energy range of the quanta giving rise to the radiation thermal spectrum. At the peak of the spectrum  $\gamma \approx 2.82$ .

This energy is provided by the work done by the gravitational field to pull the two partners apart. We can compute this work in the static frame outside a black hole and compare it with  $\omega(r_*)$ . Using this relation, we can determine the region from which the Hawking quanta originate. This is the process we now want to implement.

Let us clarify that, in a general relativistic framework, the geodesic deviation equation does not describe the force acting on a particle moving along a geodesic. Rather, it expresses how the spacetime curvature influences two nearby geodesics, making them either diverge or converge, i.e. it effectively measures tidal effects. Therefore, we can interpret these effects as the pull of the gravitational force on particles and talk about the work done by the gravitational field only in an heuristic sense. Nevertheless, in the case considered here where the test particles have a mass much smaller than the black hole and we can neglect back-reaction effects, we expect this interpretation of the gravitational field effects to capture some relevant aspects of black hole physics. With these assumptions spelled out, let us proceed.

In the rest frame of the outgoing particle, one would see the antiparticle accelerating towards the horizon due to the tidal force. This radial acceleration in the rest frame of the particle can be computed using the geodesic deviation equation, namely

$$a^r|_{r_*} \equiv \frac{Dn^r}{D\tau^2}\bigg|_{r_*} = R^r{}_{\mu\nu\rho}u^\mu u^\nu n^\rho|_{r_*}, \quad (5)$$

where the r.h.s. is expressed in terms of the Riemann tensor components,  $n^r$  denotes the separation between the two radially infalling geodesics followed by the pair of particles and  $u^\mu = [1, 0, 0, 0]$  in the rest frame of the particle.

The separation between the particle and the antiparticle when the pair forms spontaneously (i.e. they go “on-shell”) is given by their Compton wavelength, namely  $n^\rho = [0, n^r, 0, 0]$  where  $n^r \sim \lambda_C = \hbar/mc$ , and  $m \ll M$  is the particles rest mass (from now on we shall work in units where  $\hbar = c = 1$ ). So in the end, Eq. (5) implies that the radial component of the tidal acceleration (as computed in the rest frame of the particle at coordinate  $r_*$ ) is given by <sup>4</sup>

<sup>4</sup> For computation of the acceleration in the rest frame of the particle we need the Riemann tensor in the inertial frame of the particle. One can compute the Riemann tensor in the static Schwarzschild coordinates and then boost it using the free-fall velocity of the particle as measured in the static frame. A feature of the Schwarzschild geometry is that the components of the Riemann tensor remains invariant under such a boost [16]. Thus, in (5) we have  $R_{rttr} = -2M/r^3$ .

$$a^r|_{r_*} = \frac{2M}{r_*^3} \lambda_C \quad (6)$$

Our aim is to determine the work done on the spontaneously created particle pair by the tidal force in the static frame outside the black hole. For this we need to compute the tidal force as measured by a static observer outside the black hole at the instant when the outgoing partner goes on shell. This can be achieved by considering the particle rest frame and the static observer frame as locally two inertial frames: The latter sees the particle as moving with outward velocity given by the radial component of the geodesic tangent vector  $u^r = dr/d\tau$ . Once this is known, we can derive the radial acceleration observed by the static observer by performing a boost with rapidity  $\zeta = \tanh^{-1}(u^r)$ .

We thus need to determine the instantaneous radial component of the free fall velocity of the outgoing particle when it goes on-shell. This can be computed from the geodesic equation and it is given by

$$u^r = \frac{dr}{d\tau} = \sqrt{\frac{2M}{r} \left(1 - \frac{r}{r_0}\right)}, \quad (7)$$

where  $r_0$  comes as an integration constant corresponding to the coordinate distance at which the particle velocity goes to zero. Since we are interested in the value of the radial component of the geodesic tangent vector at the instant when the outgoing particle goes on-shell and becomes an Hawking quantum which eventually reaches infinity, we can take the integration constant  $r_0 \rightarrow \infty$ , i.e. Hawking quanta can be created with zero velocity only at infinity. Hence, we get

$$u^r|_{r_*} = \sqrt{\frac{2M}{r_*}}. \quad (8)$$

We can now boost the acceleration vector  $a^\mu = (0, a^r, 0, 0)$ , where  $a^r$  given by (6), with a velocity parameter given by (8), in order to determine the tidal force in the static frame  $a_{\text{st}}^r$ . We get  $a_{\text{st}}^r = a^r \cosh(\zeta)$  so that the radial component of the force under this transformation is given by

$$F_{\text{tidal-st}}^r|_{r_*} = \frac{ma_{\text{st}}^r}{(1 - 2M/r)|_{r_*}} = \frac{m\lambda_C}{(1 - 2M/r_*)^2} \frac{2M}{r_*^3}, \quad (9)$$

where we have rescaled the mass in the rest frame by the appropriate Lorentz factor,  $(1 - 2M/r_*)^{-1}$ . Finally, using the fact that  $\lambda_C \sim 1/m$ , the magnitude of the force is given by

$$||F_{\text{tidal-st}}^r|| = \frac{2M}{r_*^3} \left(1 - \frac{r_s}{r_*}\right)^{-\frac{3}{2}}. \quad (10)$$

In analogy with the Schwinger effect, we shall now assume that the work done by the tidal force to split the

virtual pair can be approximated by the product of the force computed above with the distance over which it appears to have acted, i.e. the separation of the two Hawking quanta as they go on-shell as measured by a static observer at  $r_*$ . Given that we have assumed that the incoming Hawking quantum goes on shell as soon as it can do so, i.e. at horizon crossing, this distance will coincide with the static observer's proper distance to the horizon  $d(r_*)$ .

Therefore, the work required by the tidal force to split the pair apart is given by <sup>5</sup>

$$W_{\text{tidal}} \sim \|F_{\text{tidal-st}}^r\| d(r_*) = \frac{2M}{r_*^3} \left(1 - \frac{r_s}{r_*}\right)^{-\frac{3}{2}} d(r_*), \quad (11)$$

where  $d(r_*)$  is given by

$$\begin{aligned} d(r_*) &= \int_{r_s}^{r_*} \sqrt{g_{rr}} dr' \\ &= r_s \left( \sqrt{\alpha(\alpha-1)} + \frac{1}{2} \log \left[ \alpha \left( 1 + \sqrt{1 - \frac{1}{\alpha}} \right)^2 \right] \right), \end{aligned} \quad (12)$$

and we have defined  $\alpha \equiv r_*/r_s$ .

We can then equate this work to the total energy of the two Hawking quanta being created, namely  $W_{\text{tidal}} = 2\omega_r$ . This gives us

$$\frac{2M}{r_*^3} \left(1 - \frac{2M}{r_*}\right)^{-\frac{3}{2}} d(r_*) = \frac{\gamma}{2\pi r_s} \left(1 - \frac{2M}{r_*}\right)^{-\frac{1}{2}}. \quad (13)$$

Finally, from eq. (13) we get

$$\begin{aligned} \gamma &= \frac{2\pi}{\alpha^2} \left(1 - \frac{1}{\alpha}\right)^{-\frac{1}{2}} \\ &\cdot \left( 1 + \frac{1}{2\sqrt{\alpha^2 - \alpha}} \log \left[ \alpha \left( 1 + \sqrt{1 - \frac{1}{\alpha}} \right)^2 \right] \right). \end{aligned} \quad (14)$$

The relation between  $\gamma$  and  $\alpha$ , i.e the radial distance scaled as  $r_*/r_s$ , is better illustrated in Fig. 1. It is clear from the plot that the part of the Hawking thermal spectrum around the peak ( $\gamma \sim 2.82$ ), where most of the radiation is concentrated, corresponds to a region which extends far outside the horizon, up to around  $2r_s$  (at the peak  $r_* \approx 4.38 M$ ).

The plot above also shows how, in this tidal force derivation, the quanta with higher velocity (kinetic energy) are produced closer to the horizon. This is consistent with our analysis since the higher the initial radial velocity the stronger the Lorentz contraction of the

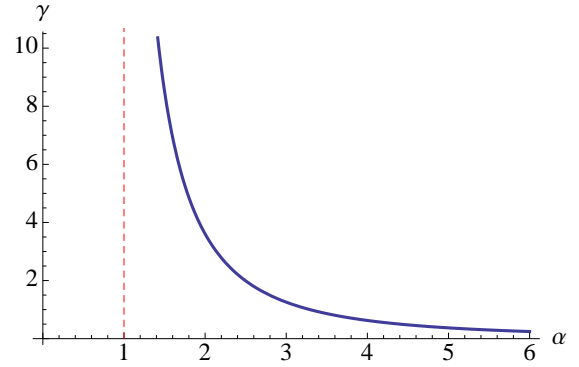


FIG. 1: This plot shows the variation of  $\gamma$  with respect to the radial distance from the center of the black hole. The red dashed line corresponds to the horizon location at  $\alpha = 1$  where the expression for the tidal force work diverges, indicating that the quanta in the far UV tail of the Hawking spectrum originate from very near the horizon.

outgoing particles distance from the horizon in their rest frame, given by  $\lambda_C$ , resulting in a shorter proper distance  $d(r_*)$  at which they are detected.

Also, by using Eq. (12) and expressing the rest of Eq. (11) in terms of  $\alpha$ , we can see that the work doable at fixed  $\alpha$  by the tidal forces scales as the inverse of the mass of the black hole so making evident that smaller holes can produce hotter particles at the same relative distance from the horizon.

Let us stress again the heuristic nature of our argument. We are considering the instantaneous value of the tidal force observed by the outgoing partner at a given coordinate distance  $r_*$  where it goes on-shell. However, we then use this instantaneous value to compute the work done by the gravitational field over a distance  $d(r_*)$ , as if the force was actually at work with the same constant value throughout the whole splitting process.

So, although the analogy with the Schwinger effect for the electron-positron pair production by an electric field may be advocated to lend support to our description of Hawking quanta production from a quantum atmosphere that extends well beyond the horizon, we now want to present a more sound analysis based on the renormalized stress energy tensor in order to confirm this picture.

### III. STRESS-ENERGY TENSOR

By analyzing the renormalized stress energy tensor (RSET) one can understand Hawking radiation in a better way as this is a local object which can help to probe the physics in the vicinity of the black hole.

<sup>5</sup> Alternatively, we could introduce a 4-vector  $\ell^\mu = (0, \ell^r, 0, 0)$ , with  $\|\ell\| = \sqrt{g_{\mu\nu} \ell^\mu \ell^\nu} = d(r_*)$ , and compute the work as  $W_{\text{tidal}} \sim g_{rr} F_{\text{tidal-st}}^r \ell^r|_{r_*}$ . This would give the same result.

### A. Computation of RSET

Let us introduce a set of globally defined affine coordinates  $U, V$  on  $\mathcal{I}_{\text{left}}^-, \mathcal{I}_{\text{right}}^-$  respectively. Restricting to the radial and time dimensions, the metric reads

$$ds^2 = C(U, V)dUdV. \quad (15)$$

In  $(1+1)$  dimensions the renormalised stress energy tensor for any massless scalar field in terms of these affine null coordinates can be easily computed using the conformal anomaly [17, 18]. The components of the RSET computed in some arbitrary vacuum state are given as:

$$\begin{aligned} \langle T_{UU} \rangle &= -\frac{1}{12\pi} C^{1/2} \partial_U^2 C^{-1/2} \\ &= \frac{1}{24\pi} \left[ \frac{C_{,UU}}{C} - \frac{3}{2} \frac{(C_{,U})^2}{C^2} \right], \end{aligned} \quad (16)$$

$$\begin{aligned} \langle T_{VV} \rangle &= -\frac{1}{12\pi} C^{1/2} \partial_V^2 C^{-1/2} \\ &= \frac{1}{24\pi} \left[ \frac{C_{,VV}}{C} - \frac{3}{2} \frac{(C_{,V})^2}{C^2} \right], \end{aligned} \quad (17)$$

$$\langle T_{UV} \rangle = \frac{RC}{96\pi} = \frac{1}{24\pi} \partial_U \partial_V \ln C, \quad (18)$$

where  $C$  is the conformal factor introduced in the above metric and  $R$  is the scalar curvature.

Now let us also introduce a null coordinate  $u$  affine on  $\mathcal{I}_{\text{right}}^+$  such that

$$U = p(u); \quad (19)$$

from this we get

$$\partial_U = \dot{p}^{-1} \partial_u. \quad (20)$$

In terms of the set  $(u, V)$ , the metric reads

$$ds^2 = \bar{C}(u, V) du dV, \quad (21)$$

with

$$\bar{C}(u, V) = \dot{p}(u) C(U, V). \quad (22)$$

Assuming that the observer is always outside the collapsing star,  $\bar{C}(u, V)$  would be the metric component of a static spacetime. In terms of this newly defined null coordinate, a simple computation shows that  $T_{UU}$  is given as

$$\langle T_{UU} \rangle = -\frac{\dot{p}^{-2}}{12\pi} \left[ \bar{C}^{1/2} \partial_u^2 \bar{C}^{-1/2} - \dot{p}^{1/2} \partial_u^2 \dot{p}^{-1/2} \right]. \quad (23)$$

Now  $T_{VV}$  will have only a static contribution if  $V = v$  but if the affine null coordinate on  $\mathcal{I}_{\text{left}}^+$  is defined as

$$V = q(v) \quad (24)$$

and we define  $C'(U, v) = \dot{q}(v) C(U, V)$ ,  $T_{VV}$  is given as

$$\langle T_{VV} \rangle = -\frac{\dot{q}^{-2}}{12\pi} \left[ C'^{1/2} \partial_v^2 C'^{-1/2} - \dot{q}^{1/2} \partial_v^2 \dot{q}^{-1/2} \right]. \quad (25)$$

As mentioned earlier  $\bar{C}(u, V)$  is the metric component of a static spacetime, so all the dynamics of the collapsing geometry is captured in the  $\dot{p}$  term of (23). In the above analysis, by using another affine null coordinate, we can differentiate between the static contribution to the RSET and that due to the the dynamics associated with the collapse [19].

### B. RSET for different vacuum states.

In order to extract physical information from the RSET, we want to compute the energy density and the flux experienced by a free falling observer (along constant Kruskal position) long after the collapse has begun in the two physically relevant states for Hawking radiation, namely the Hartle–Hawking and the Unruh states. Therefore, in this section we are going to explicitly evaluate the general expressions for the RSET components expectation values obtained above in these two cases.

Using (A2) we get the relations

$$\dot{p}(u) \equiv \partial_u p(u) = -\frac{p(u)}{2r_s}, \quad (26)$$

$$\ddot{p}(u) = \frac{p(u)}{4r_s^2} = -\frac{\dot{p}(u)}{2r_s}. \quad (27)$$

For computing the first term of (23) we can write

$$\bar{C}^{1/2} \partial_u^2 \bar{C}^{-1/2} = \frac{3}{4} \bar{C}^{-2} (\partial_u \bar{C})^2 - \frac{1}{2} \bar{C}^{-1} \partial_u^2 \bar{C}. \quad (28)$$

Using the metric conformal factor  $C$  from (A1) we get

$$\begin{aligned} \partial_u \bar{C} &= \partial_u [\dot{p}(u) C] = \ddot{p} C + \dot{p} \partial_u C \\ &= \dot{p}(u) \left( -\frac{1}{2r_s} + \frac{r^2 - r_s^2}{2r^2 r_s} \right) C \\ &= -\frac{r_s}{2r^2} \bar{C}, \end{aligned} \quad (29)$$

and

$$\partial_u^2 \bar{C} = -\frac{1}{2} r_s \partial_u \left( \frac{\bar{C}}{r^2} \right) = \frac{r_s^2}{4r^4} \bar{C} - \frac{1}{2} \frac{r_s f(r) \bar{C}}{r^3}. \quad (30)$$

Using the above relation in (28) we have

$$\begin{aligned} \bar{C}^{1/2} \partial_u^2 \bar{C}^{-1/2} &= \frac{3}{4} \bar{C}^{-2} \left[ \frac{r_s^2}{4r^4} \bar{C}^2 \right] \\ &- \frac{1}{2} \bar{C}^{-1} \left[ \frac{r_s^2}{4r^4} \bar{C} - \frac{1}{2} \frac{r_s f(r) \bar{C}}{r^3} \right] \\ &= -\frac{3}{16} \frac{r_s^2}{r^4} + \frac{r_s}{4r^3} - \frac{3}{4} \frac{M^2}{r^4} + \frac{M}{2r^3}, \end{aligned} \quad (31)$$

where  $f(r)$  is given in (A8) and we used  $r_s = 2M$  in the last step. For the second term on the r.h.s. of (23), we have

$$\dot{p}^{1/2} \partial_u^2 \dot{p}^{-1/2} = -\frac{\dot{p}^{1/2}}{2} \partial_u \left( \frac{\ddot{p}}{\dot{p}^{3/2}} \right) = \frac{1}{(8M)^2}. \quad (32)$$



We are now ready to compute explicitly the expectation value of the different RSET components for the Hartle–Hawking ( $|H\rangle$ ) and Unruh ( $|U\rangle$ ) states.

We can start by observing that for the  $T_{UU}$  and  $T_{UV}$  components, the expectation values are the same in the two vacuum states [18]. Therefore, in the following we simply denote

$$\langle T_{UU} \rangle \equiv \langle H|T_{UU}|H \rangle = \langle U|T_{UU}|U \rangle, \quad (33)$$

$$\langle T_{UV} \rangle \equiv \langle H|T_{UV}|H \rangle = \langle U|T_{UV}|U \rangle. \quad (34)$$

By means of (31), (32),  $\langle T_{UU} \rangle$  is given by

$$\begin{aligned} \langle T_{UU} \rangle &= \frac{\dot{p}^{-2}}{24\pi} \left[ \frac{3}{2} \frac{M^2}{r^4} - \frac{M}{r^3} + \frac{1}{32M^2} \right] \\ &= (768\pi M^2)^{-1} \frac{V^2}{4r^2} e^{-r/M} \left[ 1 + \frac{4M}{r} + \frac{12M^2}{r^2} \right]. \end{aligned} \quad (35)$$

To compute  $\langle T_{UV} \rangle$  we use (18), from which

$$\begin{aligned} \langle T_{UV} \rangle &= \frac{1}{24\pi} \partial_U \partial_V \ln C = \frac{1}{24\pi} (\dot{p}\dot{q})^{-1} \partial_u \partial_v \ln C \\ &= -\frac{1}{96\pi} (\dot{p}\dot{q})^{-1} C \partial_r^2 C. \end{aligned} \quad (36)$$

Using  $C(t, r)$  from (A1) and the exact values of  $q(u)$  and  $p(v)$ , we get

$$\langle T_{UV} \rangle = -\frac{M^2}{12\pi r^4} e^{-r/2M}. \quad (37)$$

On the other hand, the dependence of  $\langle T_{VV} \rangle$  on the state in which we are computing the expectation value is important. For the Hartle–Hawking state (eternal black hole scenario, non-singular vacuum state in both past and future horizons) in Kruskal coordinates the modes are given by  $e^{-i\omega U}$ ,  $e^{-i\omega V}$ , where we defined  $V$  as

$$V \equiv q(v) = 2r_s e^{v/2r_s}. \quad (38)$$

Using this definition of  $V$  we can proceed in a similar way as for the computation of  $\langle T_{UU} \rangle$ . From (25), we obtain

$$\begin{aligned} \langle H|T_{VV}|H \rangle &= \frac{\dot{q}^{-2}}{24\pi} \left[ \frac{3}{2} \frac{M^2}{r^4} - \frac{M}{r^3} + \frac{1}{32M^2} \right] \\ &= (768\pi M^2)^{-1} \frac{U^2}{4r^2} e^{-\frac{r}{M}} \left[ 1 + \frac{4M}{r} + \frac{12M^2}{r^2} \right]. \end{aligned} \quad (39)$$

For the Unruh state in Kruskal coordinates, the modes are given by  $e^{-i\omega U}$ ,  $e^{-i\omega v}$  and there is no regularization condition imposed in the past horizon. The expectation value of the  $T_{VV}$  component can be obtained from the relation

$$\langle U|T_{VV}|U \rangle = 16M^2 \dot{q}^{-2} \langle U|T_{vv}|U \rangle, \quad (40)$$

where  $\langle U|T_{vv}|U \rangle$  can be computed from

$$\langle U|T_{vv}|U \rangle = -\frac{1}{12\pi} f(r)^{1/2} \partial_v^2 f(r)^{-1/2} \quad (41)$$

using  $f(r) = (1 - \frac{2M}{r})$ , as follows from the metric of a black hole in static Schwarzschild coordinates. We have

$$\langle U|T_{vv}|U \rangle = \frac{1}{24\pi} \left[ \frac{3M^2}{2r^4} - \frac{M}{r^3} \right], \quad (42)$$

and from (40) we get

$$\langle U|T_{VV}|U \rangle = \frac{1}{6\pi} \frac{M^2}{V^2} \left[ \frac{3M^2}{2r^4} - \frac{M}{r^3} \right]. \quad (43)$$

### C. Energy density

We now have all the ingredients to extract physical information from the RSET. Let us first analyze the energy density as measured in the frame of an observer moving along fixed position in Kruskal coordinates.

Let us consider an observer at a given Kruskal position with 2-velocity  $v^\mu = C^{-1/2}(1, 0)$  (in  $[T, X]$  coordinates). The energy density,  $\rho$ , measured by this observer for the Unruh state is given by

$$\begin{aligned} \rho &= \langle U|T_{\mu\nu}|U \rangle v^\mu v^\nu = C^{-1} \langle U|T_{TT}|U \rangle \\ &= C^{-1} \langle U|T_{VV} + T_{UU} + 2T_{UV}|U \rangle. \end{aligned} \quad (44)$$

Using (35), (37), (43) we can compute the energy density exactly and we plot it in FIG. 2 (where  $\alpha \equiv r/r_s$ ).

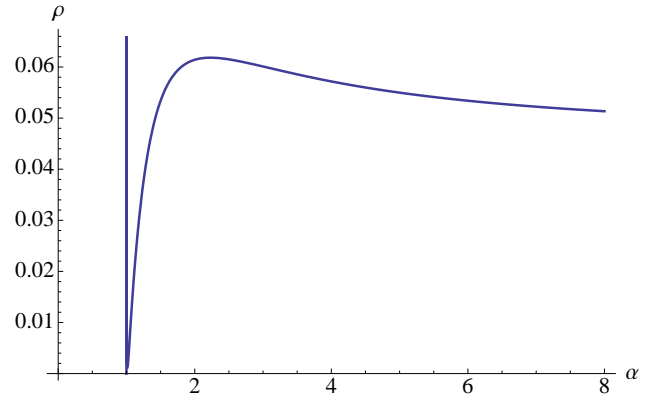


FIG. 2: Plot of the energy density as a function of the radial distance from the centre of the black hole in Unruh state.

We see that the energy density blows up at the horizon ( $r = 2M$ ) since we are computing the energy density as observed by a free falling observer in the Unruh state which is well known to be ill defined on the past horizon. The significant thing for us is the peak in the distribution of  $\rho$  that is obtained outside the horizon which is at  $r \approx 4.32 M$ . Quite in agreement with our heuristic prediction based on the gravitational analogue of the Schwinger effect.

To get a non-singular energy density for the free falling observer we should consider the Hartle–Hawking state.

This is given by

$$\begin{aligned}\rho &= \langle H|T_{\mu\nu}|H\rangle v^\mu v^\nu = C^{-1}\langle H|T_{TT}|U\rangle \\ &= C^{-1}\langle H|T_{VV} + T_{UU} + 2T_{UV}|H\rangle.\end{aligned}\quad (45)$$

Using the expectation values given in (35), (37), (39), we can plot the energy density (45) with respect to radial distance parametrized by  $\alpha$ . This is shown in FIG. 3, where we see a similar nature of the distribution with a peak outside the horizon; however, as expected, in this case the energy density is regular everywhere. Remarkably, the peak is located at  $r \approx 4.37M$ , in close agreement with our heuristic findings.

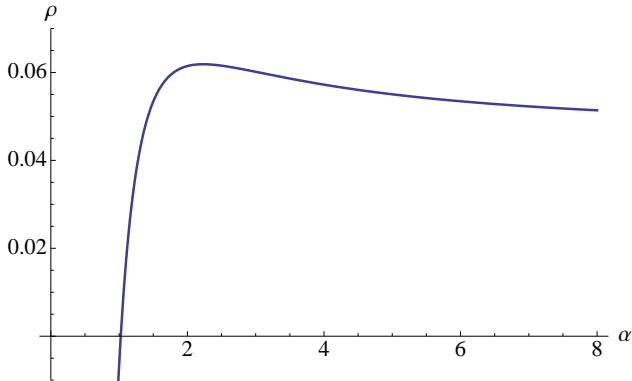


FIG. 3: Plot of the variation of energy density computed in Hartle–Hawking state with respect to the radial distance from the centre of the black hole at fixed time measured in the static frame. Notice that close to the horizon the energy density is negative.

These results strongly support our previous claim that the radiation density is maximized in a region outside the horizon. We now show that a similar behavior with a peak away from the horizon is exhibited also by the flux part of the RSET.

#### D. Flux

The flux of the Hawking radiation in the Unruh vacuum is given by <sup>6</sup>

$$F = \langle U|T_{\mu\nu}|U\rangle v^\mu z^\nu, \quad (46)$$

where  $v^\mu$  is the velocity of the observer and  $z^\nu$  is the contravariant component of the normal to the observer. Let us consider a static observer at fixed distance in a Kruskal frame with  $v^\mu = C^{-1/2}[1, 0]$  and indicate the normal vector as  $z^\nu = [A, B]$ . The latter has to satisfy the following conditions

$$g_{\mu\nu}z^\mu z^\nu = -1, \quad z^\mu v_\mu = 0. \quad (47)$$

Using the second relation we get  $A = 0$  and from the first relation we get  $B = C^{-1/2}$ . Therefore,  $z^\nu = C^{-1/2}[0, 1]$ .

Using these expressions for  $v^\mu, z^\nu$ , we get

$$F = C^{-1}\langle U|T_{TX}|U\rangle = C^{-1}\langle U|[T_{VV} - T_{UU}]|U\rangle. \quad (48)$$

Plugging in the expectation values (35), (43) found above, we can plot the flux as a function of  $\alpha$ . This is shown in FIG. 4. We see that the flux has a maximum at

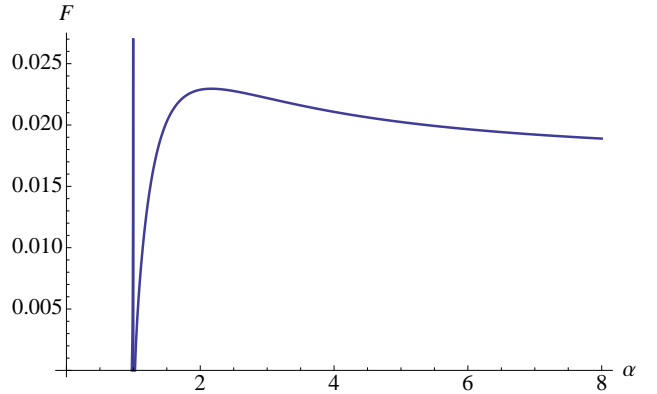


FIG. 4: This plot shows the variation of the flux of Hawking radiation with respect to the radial distance as measured by an observer in the Unruh state.

$r = 4.32M$  and most of the contribution to the Hawking radiation comes from a region between the horizon and  $r \approx 6M$ . Let us notice that in the Unruh state the flux diverges at  $r_s$ , again because of the divergence of  $T_{UU}$  on the past horizon.

#### IV. SUMMARY AND DISCUSSION

It has been widely believed that Hawking radiation originates from the excitations close to the horizon and this eventually suggested some drastic modification of the states in the near horizon regime as a resolution to the information loss paradox [5, 20–22]. One of the primary reasons for such an argument is based on the way Hawking did his original calculation, tracing back the modes all the way from future infinity to the past null infinity through the collapsing matter so that one has a vacuum state at the horizon for a free-falling observers.

The other disturbing feature about this argument is, when the modes are traced back they become highly blueshifted near the horizon and we are not well aware of the laws of physics in such high transplanckian domain. Some resolutions to the above problem has been proposed several time in the literature [23–25] but they all demand some challenging modification to our present knowledge of gravitation or quantum field theory.

Let us stress, however, that the UV departures from Lorentz invariance through the introduction of a fundamental cutoff postulated in [26, 27] are relevant only very close to the horizon for large black holes (in units of the

<sup>6</sup> In the Hartle–Hawking vacuum the flux vanishes due to the thermal equilibrium of the state.

Lorentz breaking scale). Hence, even contemplating such scenario, our analysis in section III would be basically unchanged and unaffected away from the horizon, as also stressed in the similar analysis carried out in [28].

In this paper we have shown evidence that the Hawking quanta originate from a region which is far outside the horizon, which can be called a black hole *atmosphere*. More precisely, from the plots of the energy density and the flux in the Unruh state we get a maximum at  $r \approx 4.32M$ , for the energy density in the Hartle–Hawking state the peak is at  $r \approx 4.37M$ . This is strikingly close to our previous finding for an origin at about  $r \approx 4.38M$  for the peak of the thermal spectrum using the heuristic argument based on tidal forces. By large this is also in agreement with some previous claims using various other methods, such as calculating the effective radius of a radiating body using the Stefan–Boltzmann law or computing the effective Tolman temperature [10, 29, 30], as well as in close correspondence with the results of the study of the stress-energy tensor in the Boulware vacuum of [31].

If the radiation has a long distance origin then we might not need to worry about the transplanckian issue at the horizon. Moreover, concerning the fundamental issue of unitarity of black hole evaporation, this result suggests to consider some effect operational at this new scale in order to eventually restore unitarity of Hawking radiation. A possible scenario is the one of non-violent nonlocality advocated in [32, 33]. We hope that the present contribution will stimulate further investigations in these directions.

## V. ACKNOWLEDGMENT

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## Appendix A: Kruskal frame.

We want to examine the components of the RSET in a globally well defined coordinate system free of any patho-

logical behavior (other than a true curvature singularity, like in the center of a black hole). For this purpose the Kruskal coordinate frame is an appropriate choice. The Kruskal metric is given as

$$ds^2 = \frac{r_s}{r} e^{-r/r_s} dU dV, \quad (\text{A1})$$

where  $r_s$  is the radius of the event horizon. For this coordinate system we have

$$U = p(u) = -2r_s e^{-u/2r_s}, \quad (\text{A2})$$

$$V = q(v) = 2r_s e^{v/2r_s}. \quad (\text{A3})$$

The affine null coordinate  $u, v$  in terms of radial distance from the centre of the black hole, “ $r$ ”, and time, “ $t$ ”, as measured by a static observer is given as

$$u = t - r_* = t - \left[ r + r_s \ln \left( \frac{r}{r_s} - 1 \right) \right], \quad (\text{A4})$$

$$v = t + r_* = t + \left[ r + r_s \ln \left( \frac{r}{r_s} - 1 \right) \right]. \quad (\text{A5})$$

also

$$\partial_u = \frac{\partial r_*}{\partial u} \partial_{r_*} = -\frac{1}{2} \partial_{r_*} = -\frac{1}{2} f(r) \partial_r, \quad (\text{A6})$$

$$\partial_v = \frac{\partial r_*}{\partial v} \partial_{r_*} = \frac{1}{2} \partial_{r_*} = \frac{1}{2} f(r) \partial_r. \quad (\text{A7})$$

where we used

$$\frac{dr_*}{dr} = [f(r)]^{-1} = \left( 1 - \frac{r_s}{r} \right)^{-1}. \quad (\text{A8})$$

We can also define a set of time like and radial coordinates  $(T, X)$  as

$$T = \frac{1}{2}(V + U), X = \frac{1}{2}(V - U). \quad (\text{A9})$$

Using this metric (A1) is given as

$$ds^2 = \frac{r_s}{r} e^{-r/r_s} (dT^2 - dX^2). \quad (\text{A10})$$

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- [1] S. W. Hawking, “Particle creation by black holes,” *Comm. Math. Phys.* **43**, 199 (1975).
  - [2] S. W. Hawking, “The unpredictability of quantum gravity,” *Comm. Math. Phys.* **87**, 395 (1982).
  - [3] D. N. Page, “Information in black hole radiation,” *Phys. Rev. Lett.* **71**, 3743 (1993), hep-th/9306083.
  - [4] S. B. Giddings, “Black holes and massive remnants,” *Phys. Rev.* **D46**, 1347 (1992), hep-th/9203059.
  - [5] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, “Black Holes: Complementarity or Firewalls?,” *JHEP*

**02**, 062 (2013), 1207.3123.

- [6] D. Pranzetti, “Radiation from quantum weakly dynamical horizons in LQG,” *Phys. Rev. Lett.* **109**, 011301 (2012), 1204.0702.
- [7] D. Pranzetti, “Dynamical evaporation of quantum horizons,” *Class. Quant. Grav.* **30**, 165004 (2013), 1211.2702.
- [8] W. G. Unruh, “Origin of the particles in black-hole evaporation,” *Phys. Rev. D* **15**, 365 (1977).
- [9] R. Parentani, “From vacuum fluctuations across an event horizon to long distance correlations,” *Phys. Rev.* **D82**,



- 025008 (2010), 1003.3625.
- [10] S. B. Giddings, “Hawking radiation, the Stefan-Boltzmann law, and unitarization,” *Phys. Lett. B* **754**, 39 (2016), 1511.08221.
  - [11] M. K. Parikh and F. Wilczek, “Hawking radiation as tunneling,” *Phys. Rev. Lett.* **85**, 5042 (2000), hep-th/9907001.
  - [12] S. Hawking and W. Israel, *General Relativity; an Einstein Centenary Survey* (Cambridge University Press, 1979), ISBN 9780521222853.
  - [13] R. J. Adler, P. Chen, and D. I. Santiago, “The Generalized uncertainty principle and black hole remnants,” *Gen. Rel. Grav.* **33**, 2101 (2001), gr-qc/0106080.
  - [14] L. Parker, *The Production of Elementary Particles by Strong Gravitational Fields* (Springer US, Boston, MA, 1977), pp. 107–226, ISBN 978-1-4684-2343-3.
  - [15] J. S. Schwinger, “On gauge invariance and vacuum polarization,” *Phys. Rev.* **82**, 664 (1951).
  - [16] C. Misner, K. Thorne, and J. Wheeler, *Gravitation*, no. pt. 3 in *Gravitation* (W. H. Freeman, 1973), ISBN 9780716703440.
  - [17] P. C. W. Davies, S. A. Fulling, and W. G. Unruh, “Energy-momentum tensor near an evaporating black hole,” *Phys. Rev. D* **13**, 2720 (1976).
  - [18] N. Birrell and P. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, 1984), ISBN 9780521278584.
  - [19] C. Barcelo, S. Liberati, S. Sonego, and M. Visser, “Fate of gravitational collapse in semiclassical gravity,” *Phys. Rev. D* **77**, 044032 (2008), 0712.1130.
  - [20] K. Papadodimas and S. Raju, “An Infalling Observer in AdS/CFT,” *JHEP* **10**, 212 (2013), 1211.6767.
  - [21] S. L. Braunstein, S. Pirandola, and K. Życzkowski, “Better Late than Never: Information Retrieval from Black Holes,” *Phys. Rev. Lett.* **110**, 101301 (2013), 0907.1190.
  - [22] A. Almheiri, D. Marolf, J. Polchinski, D. Stanford, and J. Sully, “An Apologia for Firewalls,” *JHEP* **09**, 018 (2013), 1304.6483.
  - [23] W. G. Unruh, “Sonic analogue of black holes and the effects of high frequencies on black hole evaporation,” *Phys. Rev. D* **51**, 2827 (1995).
  - [24] S. Corley and T. Jacobson, “Hawking spectrum and high frequency dispersion,” *Phys. Rev. D* **54**, 1568 (1996), hep-th/9601073.
  - [25] T. Jacobson, “Black-hole evaporation and ultrashort distances,” *Phys. Rev. D* **44**, 1731 (1991).
  - [26] W. G. Unruh, “Sonic analog of black holes and the effects of high frequencies on black hole evaporation,” *Phys. Rev. D* **51**, 2827 (1995).
  - [27] T. Jacobson, “Black hole evaporation and ultrashort distances,” *Phys. Rev. D* **44**, 1731 (1991).
  - [28] R. Brout, S. Massar, R. Parentani, and P. Spindel, “Hawking radiation without transPlanckian frequencies,” *Phys. Rev. D* **52**, 4559 (1995), hep-th/9506121.
  - [29] S. Hod, “Hawking radiation and the Stefan-Boltzmann law: The effective radius of the black-hole quantum atmosphere,” *Phys. Lett. B* **757**, 121 (2016), 1607.02510.
  - [30] M. Eune, Y. Gim, and W. Kim, “Effective Tolman temperature induced by trace anomaly,” (2015), 1511.09135.
  - [31] R. Parentani and R. Brout, “Physical interpretation of black hole evaporation as a vacuum instability,” *Int. J. Mod. Phys. D* **1**, 169 (1992).
  - [32] S. B. Giddings, “Black holes, quantum information, and unitary evolution,” *Phys. Rev. D* **85**, 124063 (2012), 1201.1037.
  - [33] S. B. Giddings, “Nonviolent nonlocality,” *Phys. Rev. D* **88**, 064023 (2013), 1211.7070.